MATHEMATICAL OPERATIONS

I. Exponents (see also Appendix A1.1)

A. Properties

1. $a^m a^n = a^{m+n}$

2. $(a^m)/(a^n) = a^{m-n}$

3. $(a^m)^n = a^{mn}$

4. $(ab)^n = a^n b^n$

B. Examples

1. $2.40\times10^{-1} + 4.5\times10^{-2} = 2.85\times10^{-1}$

2. $(2.0\times10^{6})\times(3.3\times10^{-2}) = 6.6\times10^{4}$

3. $(3\times10^{6})^{2} = 9\times10^{12}$

4. $(1.6\times10^{9})^{½} = 4.0\times10^{4}$; note that it can be useful to think of this problem as:

   $(1.6\times10^{9})^{½} = (16\times10^{8})^{½} = (16)^{½}(10^{8})^{½} = 4.0\times10^{4}$

5. $(4.8\times10^{-3})/(2.0\times10^{-6}) = 2.4\times10^{3}$

II. Logarithms (see also Appendix A.1.2)

A. Base 10 (common logarithms)

1. if log(x) = n, then $x = 10^n$; or $\log(10^n) = n$

2. $\log(1000) = \log(10^3) = 3$ (note problem of significant figures, and see below)

3. The log of a number less than 1 is negative

4. The log of a number greater than 1 is positive

5. The log of 1 is 0

6. The log of zero or a negative number is undefined (some sophisticated calculators may return a complex number involving the square root of -1)

7. Items 3-6 can be summarized as follows: log(1) = 0; log(n) < 0 if n < 1;

   log(n) > 0 if n > 1; log(n) is undefined if $n \leq 0$

8. Useful relationships that follow from the properties of exponents include;

   a. $\log(a\times b) = \log(a) + \log(b)$

   b. $\log(a/b) = \log(a) - \log(b)$

   c. $\log(a^b) = b\times\log(a)$

B. Examples

1. $\log(1000) = \log(10^3) = 3$

2. $\log(0.001) = \log(10^{-3}) = -3$

3. $\log(0.0422) = \log(4.22\times10^{-2}) = \log(4.22) + \log(10^{-2}) = 0.625 - 2 = -1.375$

4. $\log(422) = \log(4.22\times10^{2}) = \log(4.22) + \log(10^{2}) = 0.625 + 2 = 2.625$

5. If a = 100 and b = 0.001, $\log(a\times b) = \log(a) + \log(b) = 2 + -3 = -1$, and

   $\log(a/b) = \log(a) - \log(b) = 2 - (-3) = 5$.

C. Antilogarithm (calculators often label this key 10^x, for good reason)

$\text{antilog}(x) = 10^x$

$\text{antilog}(2.625) = 10^{2.625} = 422$

$\text{antilog}(-1.375) = 10^{-1.375} = 0.0422$

C. Base e or natural logarithms, ln(x)

if $\ln(x) = n$, then $x = e^n$, where $e = 2.718282$; or $\ln(e^n) = n$

$\ln(x) = 2.303\log(x)$, where $2.303 = \ln(10)$. This can be seen by considering $x = 10^n$, so $\log(x) = n$. $\ln(x) = \ln(10^n) = n\times\ln(10)$ from II.A.8.c. But $n = \log(x)$ so $\ln(x) = \ln(10)\times\log(x)$.

III. Significant Figures (see also Appendix A.1.6)

A. Examples

3.1416; five significant figures

0.00023; two significant figures (2.3×10^{-4})

0.860; three significant figures (8.6×10^{-1} is different than 8.60×10^{-1})

B. Basic Operations

8.65 - 0.024 = 8.63

5.24/1.1 = 5.7

1/9 = 0.11 in spite of what your calculator may say

C. Functions

If y is a function of x, *i.e.* y = f(x), the number of significant figures in y need not equal the number of significant figures in x. For example, $\log(422) = 2.625$; $10^{2.62} = 417$, $10^{2.63} = 427$, $10^{2.625} = 421.7$.

IV. Quadratic Equations (see also Appendix A.1.4)

$ax^2 + bx + c = 0$ represents a general quadratic equation, where a, b, c can be positive, negative or zero. There are always two solutions to this equation, given by $x = [-b \pm (b^2 - 4ac)^{½]}/2a$. Note that $b^2 - 4ac$ must be positive to give real answers, otherwise the answers will be complex, involving the square root of -1. Some calculators are programed to do this rather automatically, and we will discuss various approximate methods in class.

V. Temperature Scales

Celsius, °T (Centigrade)

Fahrenheit, °F

Kelvin, K (note the degree sign, °, is NOT used with Kelvin temperatures)

K = °C + 273

°C = 5(°F - 32)/9

°F = 9(°C)/5 + 32